ISSN 2229-5518

ANALYSIS OF RECONSTRUCTED IMAGES USING COMPRESSIVE SENSING

Ms.Ambily Joseph, Ms.Jaini Sara Babu, Dr. K.P Soman
Computational Engineering and Networking
Amrita School of Engineering, Coimbatore
josephambily@gmail.com, jainibabu@yahoo.co.in

Abstract – Traditionally image reconstruction is done by performing Fast Fourier Transform (FFT). But recently there has been growing interest in using compressive sensing (CS) to perform image reconstruction.In compressive sensing, the main property of signal-Sparsity is explored for reconstruction purposes.In this paper, for image reconstruction, various optimization techniques like L1 optimization, Total Variation (TV) Minimization and Split-Bregman Optimization is used. Among these, the Split-Bregman reconstruction algorithm shows good performances. This is proved by the analysis of reconstructed images using the quality measures such as Peak Signal-to-Noise Ratio (PSNR) and Normalised Absolute Error (NAE).

Index Terms – FFT, Compressive Sensing, L1, TV, Split-Bregman, Quality Measures

1 INTRODUCTION

Compressive Sensing (CS) based image reconstruction offers a great advantage over traditional transform coding methods.In traditional method, initially sampling process is followed by compression which consist of three level.In this, first we must consider N samples.Second,the transform coefficients of these N samples is computed. Later on we will have to discard some of it.Third, the location of these transform coefficients is encoded.In this, encoding the position of large coefficients is a crucial problem faced by an encoder [1].

Over the recent few years, to compress the signals below Nyquist rate a new method called compressive sensing (CS) has been discovered. According to CS an unknown signal can be recovered from a small number of random measurements by sparsity-promoting nonlinear recovery algorithms .Recently, many recovery algorithms have been proposed such as L1 optimization, Total Variation (TV) Minimization and Split-Bregman Optimization.However in L1 optimization, for the sampling of an image, the L1 reconstruction quality is not quite satisfying as image does not have a strictly sparse representation in another transformation domain.So we go for Total Variation(TV) Minimization of the image.The Split-Bregman method was recently proposed and it can effectively solve general L1-Regularised Optimization problems with multiple L1-Regularised terms while the linearised Bregman algorithm and fixed point continuation methods fail.

It is necessary to establish image quality measures to compare the different recovery algorithms mentioned above.Image quality measures are evaluated for a reconstructed image by comparing it with its original image.Here image quality measure is done with Peak Signal-to-Noise Ratio (PSNR) and Normalised Absolute Error (NAE).This is made on three test images for L1, TV and Split-Bregman methods with 30 percent, 50 percent and 80 percent fourier coefficients as sparsity values.

The organization of paper is as follows. Section 2 describes FFT method and three other reconstruction methods based on compressive sensing. Image quality measures PSNR and NAE are discussed in section 3. Section 4 gives results of the reconstruction algorithms and a comparison table them based on image quality measures.Also a brief description on parameters of image transmission is given. Finally the work is concluded in section 5.

2 IMAGE RECONSTRUCTION METHODS

2.1 Fast FourierTransform

Image reconstruction can be performed traditionally with FFT and also by newly emerging method Compressive Sensing .Compressive Sensing is based on the recent understanding that a small collection of non adaptive linear measurements of a compressible signal or image contains enough information for reconstruction and processing [1]. This is a great advantage over traditional reconstruction method such as FFT method. The FFT based image reconstruction is done by decomposing the image into its sine and cosine components and then reconstructing it by taking inverse Fourier transform (IFFT). Fourier transformation is done on the input image by:

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vx/N)} \tag{1}$$

where $f(x,y)$ represents each pixel on the image at x and y coordinates.The above Fourier transform must be evaluated for values of the discrete variables u and v in the ranges $u=0,1.....M-1$ and $v=0,1.....N-1$. After performing FFT, the Fourier coefficients are normalised. Now the absolute value of these coefficients is taken. In order to retain all our fine details while reconstruction, we find an appropriate threshold value where there is a major variation in these Fourier coefficients. Finally perform inverse FFT (IFFT) by using less number of Fourier coefficients which are greater than the threshold of encoding the locations of the large valued coefficients value. The problem in FFT method is that the encoder faces the overhead along with the values for data transmission in real time applications.

2.2 Compressive Sensing Algorithms

As an alternative, algorithms with Compressive Sensing (CS) principle are preferred. In compressive sensing, sparsity, which can also be interpreted as compressibility of signal, is an important property in the data acquisition process. Though what differs most from Compressive sensing and classical data acquisition technique is that compressive sensing explores the sparsity of signals i.e. that it directly samples the data into a compressed form.Consider any signal $x(t)\in R^N$

$$x(t)=\sum_{i=1}^{N} s_i\psi_i(t), t=1,2,....N \tag{2}$$

With appropriate orthonormal basis $\psi_i(t)$, a signal can be represented as a superposition of wavelets, sinusoids, and so on [2-4]. Using matrix notations signal can be represented as $x=\psi s$

$$x=\psi s \tag{3}$$

Where x and s are column vectors of length $i=1,2,....N$ by N matrix with $\psi_i(t)$, $i=1,2,....N$ as columns. Since ψ is also orthogonal, s can obtain from x as

$$s=\psi^* x \tag{4}$$

So s is a K- sparse signal if there is only K nonzero values in s with $K \ll N$. Compressive sensing requires multiplying the original signal x by a M by N matrix to obtain the M measurements.To ensure the stability of CS, a measurement matrix ϕ is needed which is incoherent with the sparsifying basis ψ. If the original signal x is a length- N vector, the M x 1 vector y can be obtained from

$$y=\phi x \tag{5}$$

If signal x has a K-sparse representation s as in equation 3, with $M \ll N$ measurement $(M=cK\log(N/K)), x$ can be reconstructed exactly.We can combine equation 3 and 5 as

$$y=\theta s, (\theta=\phi\psi) \tag{6}$$

Now from the y measurements, the signal x can be reconstructed using any optimization problem like L1 optimization as

$$\min \| s' \|_{l1}, s.t: \theta s'=y \tag{7}$$

After solving s′ from the minimization problem, x is obtained with the help of equation 3. However, for the sampling of an image, the L1 reconstruction quality is not quite satisfying as image does not have a strictly sparse representation in another transformation domain.Since gradient is sparse for an image to recover an image x, it is better to minimize the total variation.

$$\min \| X' \|_{TV}, s.t: \phi x' = y \tag{8}$$

Where $$\| X' \|_{TV}=\sum_{i,j} |(\nabla X')_{i,j}| \tag{9}$$

The algorithm used here is TVAL3 (TV Minimization by Augmented Lagrangian and Alternating Direction Algorithms). This algorithm framework is a lagrangian multiplier method applied to a particular augmented Lagrangian function.

Split-Bregman algorithm is the next improved compressive sensing method which can solve a very broad class of L1 regularised problems.

$while \| p^k - p^{k-1} \|_2 > tol$

$\quad for\ i=1\ to\ N$

$\qquad p^{k+1} = \min_p H(p) + (\lambda/2) \| d^k - \phi(p) - b^k \|_2^2$

$\qquad d^{k+1} = \min_d |d| + (\lambda/2) \| d - \phi(p^{k+1}) - b^k \|_2^2$

$\quad end$

$\qquad b^{k+1} = b^k + (\phi(p^{k+1}) - d^{k+1})$

end

Here p represents the original image and b a vector.Convergence speed is high for Split-Bregman method and hence it is easy to code.

3 Image Quality Measures

After performing the optimization algorithms discussed above, two main quality metrics were taken such as Peak Signal-to-Noise Ratio (PSNR) and Normalized Absolute Error (NAE).

3.1 Peak Signal-to-Noise (PSNR)

Larger SNR and PSNR indicate a smaller difference between the original (without noise) and reconstructed image. This is the most widely used objective image quality/ distortion measure. The main advantage of this measure is ease of computation but it does not reflect perceptual quality. An important property of PSNR is that a slight spatial shift of an image can cause a large numerical distortion but no visual distortion and conversely a small average distortion can result in a damaging visual artifact, if all the error is concentrated in a small important region. This metric neglects global and composite error PSNR is calculated using following equation:

$$PSNR = 20\log_{10}(N / RMSE) \quad (10)$$

Where N is the maximum pixel value of the image and MSE (Root Mean Square Error) is given by:

$$MSE = 1/mn\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j)-K(i,j)]^2 \quad (11)$$

The PSNR value approaches infinity as the MSE approaches zero; this shows that a higher PSNR value provides a higher image quality [5]. At the other end of the scale, a small value of the PSNR implies high numerical differences between images.

3.2 Normalized Absolute Error

NAE gives normalized error value between the original and degraded image and it is given by:

$$NAE = \sum_{j=1}^{M}\sum_{k=1}^{N}|x_{j,k}-x'_{j,k}| / \sum_{j=1}^{M}\sum_{k=1}^{N}|x_{j,k}| \quad (12)$$

Here image is M X N matrix, where M denotes number of rows and N the number of columns.While the pixel coordinate in image is $(j,k), x_{j,k}$ *and* $x'_{j,k}$ denote the pixel values of original image before the compression and degraded image after the compression.

Normalised Absolute Error is a measure of how far is the decompressed image from the original image with the value of zero being the perfect fit [6].Large value of NAE indicates poor quality of the image

4 Results and Discussion

In this section, a study and comparison between different image reconstructions methods are made with the help of two test images. This helps us to study the reconstruction methods more clearly.

Here an input image “Boat” of size 256 X 256 is taken.At first, a thresholding method for image reconstruction using FFT is performed on a total of 65536 pixels.Inorder to retain all fine details while reconstruction a thresholding is performed next. Here a threshold value of 0.1 is taken. Next inverse DFT is performed by using Fourier coefficients which are greater than the above threshold value.

The above method guarantees that the largest Fourier values are not lost. Only fewer measurements are needed for reconstructing the image. i.e. here it is 3004 measurements. Now, consider the above methodology in a real time environment. In that case, for reconstruction, at the receiver side it requires random significance map along with non zero Fourier coefficients. Here the location of the fourier coefficients is the random significance map.Random significance map consist of zeros and once, where, 1 indicate non-zero locations of Fourier coefficients and zeros indicate zero coefficient values.

Next L1 optimization is performed on the “Boat”image. Note that there are some block effects on the reconstructed image when compared with the original image. The fact that quality of the reconstruction is not quite good is because only 0.3 coefficients (30 percent fourier coefficients) is taken. As quality of the reconstructed image is not so good through L1 minimization, next, another method i.e. solving the minimized total variation (TV) of an image is used.

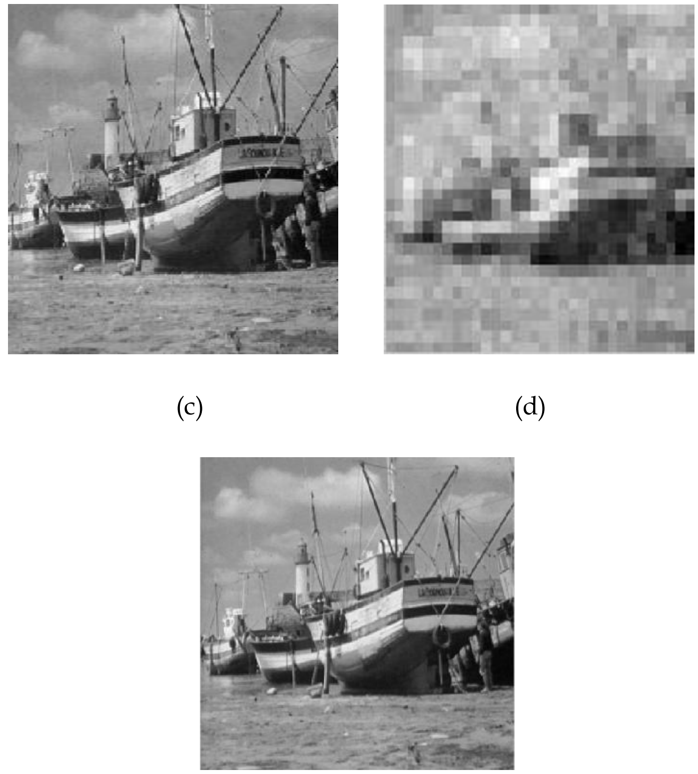
By solving the TV minimization we get a better result which is illustrated in Fig (f). With the better property of sparsity, which is of main importance for compressive sensing, a reconstructed image of good quality is obtained by solving the TV minimization.

Next same analysis is done but in the reconstruction part instead of using inverse DFT, Split-Bregman optimization algorithm is used. A sampling matrix, with entries randomly chosen to be 1 or 0 was generated first. The compressed sensing data is then computed using DFT. Finally, the Split-Bregman algorithm was used to reconstruct the image from the sub-sampled K-Space data.

The Fig.1. (b) through (e) illustrates the above mentioned methods.From Fig.1(b),we observe that DFT method has given a better result.This is because sparsity is not considered in DFT method.In Fig.1.(d),L1 optimization is used.Note that there are some block effects on the reconstructed image when compared with the original image. The fact that quality of the reconstruction is not quite good is because only 0.3 coefficients are taken. As quality of the reconstructed image is not so good through L1 minimization, next, another method i.e. solving the minimized total variation (TV) of an image is used. By solving the TV minimization.we get a better result which is illustrated in Fig 1 (e). With the better property of sparsity, which is of main importance for compressive sensing, a reconstructed image of good quality is obtained by solving the TV minimization.

Finally,the Split-Bregman algorithm was used to reconstruct the image from sub-sampled K-space data.When Split-Bregman algorithm is used,the results are obtained with less computational time due to its iterative approach.This is shown in Fig.1.(c).In Fig.1.(c) through (e) illustration of the original image is done with sparsity k=0.3.

(a) (b)



(c) (d)

(e)

Fig 1. Reconstructed images: (a) Original image (b) DFT Method (c) Split-Bregman Method (d) L1 Minimization (e)TV Minimization.

Some of the Objective measures based on computable distortion measures such as PSNR and NAE are dealt here. A comparison of FFT, Bregman, L1 and TV methods with sparsity values 0.3, 0.5 and 0.8 is done for PSNR and NAE measures. The result is given in Table I. Larger PSNR (Peak signal -to noise ratio) indicate a smaller difference between the original and reconstructed image. For all the reconstruction algorithm mentioned above largest PSNR is obtained for image with sparsity k=0.8.Also note that for FFT method, PSNR value is same for sparsity 0.3, 0.4, 0.8. This is due to the fact that sparsity constrain is not taken into consideration here. Normalized absolute error (NAE) gives an indication of how far is the reconstructed image from the original image. Large value of NAE indicates poor quality of the image. In Table I, this is observed. Large NAE values are obtained for images with less number of measurements, i.e.images with less quality.

4.1 Parameters for Image Transmission

Transmission time: Transmission time is the time taken to transmit a packet of data from transmitter to receiver. It's measured in terms of seconds. Here, FFT method has huge transmission time because along with the non-zero Fourier coefficients it has to transmit their location also.

Power Consumption: Power consumption is the power consumed during a transmission. Power consumption always depends on size of data. If the size of data is large, then power consumption will also be large as in FFT mathod.

Channel bandwidth: Channel bandwidth refers to the set of frequencies through which transmission occur in a channel. If data rate is less than bandwidth of channel, only then a communication link is set up. Otherwise, the communication link not established. Higher bandwidth results in lower latency. Latency is defined as the time taken to transmit a data packet from a transmitter station to receiver station. So, if large bandwidth is there, data can be transmitted with less amount of time.

Computational time: To explain this term, consider an image"Boat" of size 256 X 256. For sparsity k = 0.3, CPU takes 3.52 sec for 88 iterations. For sparsity k = 0.5, CPU takes 3.42 sec for 84 iterations. For sparsity k = 0.8, CPU takes 2.96 sec for 72 iterations. From this data,it is clear that for reconstruction using more number of coefficients, the process becomes more faster and thus only less comput tional time is required.

TABLE 1
COMPARISON OF RECONSTRUCTION METHODS BASED ON PSNR AND NAE

Reconstruction Methods	M	PSNR	NAE
FFT	19660 (k=0.3)	5.3635	1.0000
	39321 (k=0.5)	5.3635	1.0000
	52428 (k=0.8)	5.3635	1.0000
Split-Bregman	19660 (k=0.3)	27.0760	0.6680
	39321 (k=0.5)	31.7240	0.5920
	52428 (k=0.8)	32.4100	0.2250
L1 Minimization	19660 (k=0.3)	5.4677	0.9957
	39321 (k=0.5)	16.5202	0.2528
	52428 (k=0.8)	17.0320	0.2222
TV Minimization	19660 (k=0.3)	25.2677	0.0829
	39321 (k=0.5)	26.1142	0.0779
	52428 (k=0.8)	28.4561	0.0486

6 .CONCLUSION

In this paper, various image reconstruction approaches like FFT method, L1 optimization, TV minimization and Split-Bregman were discussed.It is concluded that, in compressed sensing problems, Split- Bregman method is an efficient solver for many problems that are difficult to solve by other minimization problems.

ACKNOWLEDGMENT

Also the authors would like to thank the Almighty and their parents whose blessings are always with them.

REFERENCES

[1] Macro.F.Duarte,Mark A.Davenport,Dharmpal Takhar,Jason N.Laska,Ting Sun,Kevin Kelly and Richard G.Baranuik (2008),"Single Pixel Imaging via Compressive Sampling",IEEE Signal Processing Magazine March,pp.83-93.

[2] D. L. Donoho, “Compressed sensing,” IEEE Trans. Inf. Theory, vol.52, no. 4, pp. 1289–1306, Sep. 2006.

[3] E. J. Candès, “Compressive sampling,” in Proc. Int. Congr. Math.,Madrid, Spain, 2006, vol. 3, pp. ,1433-1452

[4] R. G. Baraniuk, “Compressive sensing,” IEEE Signal Processing, Mag.,vol. 24, no. 4, pp. 118–120, Jul. 2007, 124.

[5] Alain Horé, Djemel Ziou," Image quality metrics: PSNR vs. SSIM", . 2010 International Conference on Pattern Recognition.

[6] Sumathi Poobal, G.Ravindran," The Performance of Fractal Image Compression on Different Imaging Modalities Using Objetive Quality Measures, International Journal of Engineering Science and Technology (IJEST), Vol. 3 No. 1 Jan 2011.

[7] Michael B. Wakin, Jason N. Laska, Marco F. Duarte, Dror B ron Shriram Sarvotham, Dharmpal Takhar, Kevin F Kelly, Richard G. Baraniuk (2006),"An Architecture For Compressive Imaging".

[8] H.T.Kung,Tsung-Han Lin,Dario Vlah (2011). "Identifying Bad Measurements in Compressive Sensing," The First International Workshop on Security in Computers,Networking and Communica tions,978-1-4244- 9920-5 .

[9] Pradeep Nagesh and Baoxin Li (2009). "Compressive Imaging Of
[10] Color Images," Dept. of Computer Science and Engineering,Arizona State University, Tempe, AZ 85287, USA.

[11] Pradeep Sen and Soheil Darabi (2009)."A Novel Framework For Imaging Using Compressed Sensing". Advanced Graphics Lab, University of New Mexico.

[12] Zhimin Xu and Edmund Y. Lam (2010)."Hyperspectral Reconstruc tion in Biomedical Imaging Using Terahertz Systems". IEEE,978-1-4244- 5309-2 .

[13] Michael B. Wakin, Jason N. Laska, Marco F. Duarte, Dror Baron, Shriram Sarvotham,Dharmpal Takhar, Kevin F. Kelly, and Richard

G. Baraniuk . ”Compressive Imaging for Video Representation and Coding ”. Dept. of Electrical and Computer Engineering Rice University, Houston, TX, USA.